Numerical Study Of Convection And Radiation Effect In A Vertical Channel With Discrete Heat Sources

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***Abstract*— Convection and Radiation Effect in geometrical configurations like vertical channels with discrete heat sources are common in the field of electronic chip cooling. This work is aimed to numerically solve such a flow and heat transfer problem using the tools of computational fluid dynamics (CFD). A vertical channel with five protruding discrete heat sources, mounted at one side is considered. In the first phase of work, positions of heat sources are kept constant and a uniform heat flux is applied. The natural convection flow through the channel is simulated for different ranges of the thermal parameter, Grashof number. Later, the same configuration with forced convection flow is considered and solved for different ranges of Reynolds number. In the second phase, same analysis is conducted be varying geometrical parameters like width of the channel and positions of the heat sources. In the third phase, mixed convection scenario is investigated. Effect of surface radiation on the mixed convection characteristics was studied briefly in the final phase. Results are presented in the form of stream lines, isothermal surfaces and vortex structures inside the channel. Variation of Nusselt number, temperature over the heat sources and Richardson number are some of the important results of this work, which are presented and discussed in detail.**

Keywords*— Discrete heat sources, natural, forced, mixed convection, Electronics chip cooling.

I. INTRODUCTION

Every human being in existence today in the universe is one or the other way influenced the items monitored and controlled by using electronic equipments. Aerospace, agriculture, biomedical, communication, visual media etc are solemnly dependent in electronics system. Of course we are using this technology in the extreme level and at the time of its continuous usage lot of heat is produced from the system manage by electronic chips. This has to be controlled to a safe level otherwise we cannot except for a good lifetime and performance of the electronic components. To overcome these problems we have to take care in design and research so that we can reach to a foolproof system before its manufacture. Effective thermal management methods are necessary to enable improvements in performance, efficiency of circuits, reliability and lifetime of the components. In the industry, several cooling strategies have been successfully implemented or are under constant development. These methods utilize fin-fan heat sinks, natural convection, micro-channels, impinging jets, dielectric fluid immersion, heat pipes, phase change materials and thermo-electrics. The sensitivity of the subject of electronic chip cooling has created an extensive research in this field during the last decades. Several studies are present in the literature using experimental as well as computational techniques with discrete heat sources inside channels. These analysis works include a coupled study of fluid dynamics and convective heat transfer.

Bejan.et.al[1] studied the optimal distribution of discrete heat sources in an open vertical channel with natural convection. He concludes the optimal spacing between heat sources increases with the Rayleigh number. Thiago Antonini Alves.et.al[2] studied about convective cooling of three discrete heat sources in horizontal channel flow. Numerical simulations of the laminar convective heat transfer from three discrete heaters flush mounted to a single wall of a channel showed that the average Nusselt number for each heater is independent of their heat flux distribution. Comparatively, the evaluated average values of Nusselt number change with the heat flux distribution. Balaji.et.al [3] examined the three-dimensional, conjugate heat transfer model with appropriate boundary conditions. Conjugate, laminar mixed-convection cooling of ten-heated electronic components mounted on a vertical channel wall was investigated numerically and the finite-volume method has been used to solve the modeling equations. The temperature field in each component is found to be almost uniform. A good better cooling is obtained when the Reynolds number is increased and when the no-heated component is placed between two heated components.

A.Bazylak et.al.[4] Presented computational analysis of flow & heat transfer inside a horizontal enclosure with an array of distributed heat sources on the bottom wall. The heat sources are modeled as flush-mounted sources with prescribed heat flux boundary conditions. Optimum heat transfer rates and the onset of thermal instability triggering various regimes are found to be governed by the length and spacing of the sources and the width-to-height aspect ratio of the enclosure. Bhowmik.et.al.[5] reported several studies related to vertical channels with heat transfer from discrete heat sources water as the working fluid. The chip wall temperatures are sharply peaked at the leading edge of the heat source and maintain a uniform wall temperature on the chip. Temperature increases from inlet to outlet. The numerical and experimental results compare favorably to the numerical results 10% less than the experimental results from all chips.

II. METHODOLOGY

A systematic approach is adopted to study the natural, forced, mixed convection and radiation heat transfer flow inside fully open vertical channels with discrete heat sources. The two dimensional partial differential equations governing the fluid flow and heat transfer formulated in Cartesian co-ordinates are solved using a commercial software package, FLUENT based on finite volume method. The schematic showing the geometry of the problem is given in figure5. Five flush mounted discrete heat sources of same power dissipation are provided on one side of the wall.

In general, before trying to solve these governing equations directly, they are transformed into dimensionless forms using some dimensionless dependent and independent variables [3]. The important dimensionless variables come in to play in the science of natural-mixed convection are Reynolds number (Re), Nusselt number (Nu), Grashof number (Gr), Prandtl number (Pr) and Raleigh number (Ra).

$$\mathrm{Re} = \frac{\rho V L_C}{\mu},\ Nu = \frac{h L_C}{k},\ \mathrm{Pr} = \frac{\mu C_P}{k},\ \ Ra = Gr.\mathrm{Pr}$$

$$Gr = \left(\frac{1}{\nu}\right)^2 D^4 g\,\beta \left(\frac{q}{k}\right),\ \text{FOR CONSTANT WALL FLUX.}$$

The non-dimensional temperature is defined here as follows.

$$\theta = \frac{T - T_\infty}{qh/k}\text{, for constant wall flux.}$$

where, L_C is the characteristic length, C_P is the specific heat, k is the thermal conductivity of the fluid, g is the gravitational acceleration, b is the thermal expansion coefficient and n is the kinematic viscosity.

A. Generation of computational grid

GAMBIT is a pre-processor included with the FLUENT package, used to create geometry and the computational grid for the solver. In this work, fully structured grids with quadrilateral elements are used.

B. Validation of the software and methodology

In order to use a computer program for the CFD analysis of any practical problem, the validation of the software and the solution methodology is essential. Even though FLUENT is a commercial software and is already validated several times in the literature, a validation study is conducted with this work as part of a standard procedure. A classical benchmark problem; the boundary driven cavity for fluid flow [6] is selected for validation of the software. After successful validation of the software, the methodology adopted to simulate mixed convection in inclined channels with heat sources is validated by reproducing results presented by Menon et.al. [7]. In order to validate the radiation model, a benchmark problem is selected. Combined heat transfer of radiation and natural convection in a square cavity is containing participating gases under normal room conditions [8].

1. The boundary driven cavity

The boundary driven cavity is a universally accepted problem for benchmarking in the field of CFD. Geometrically it is a square enclosure and the top lid is moving in a constant speed at the x-direction. The Reynolds number is defined as $\frac{\rho v L_c}{\mu}$.

Analysis is conducted for Re=1000 using a 110x110 mesh with grading towards the walls to capture the boundary layer. Variation of the horizontal component of velocity along the vertical direction through mid-point of the cavity is compared with benchmark values and shown in figure 1. It can be seen that the present predictions are in close agreement with the reported values.

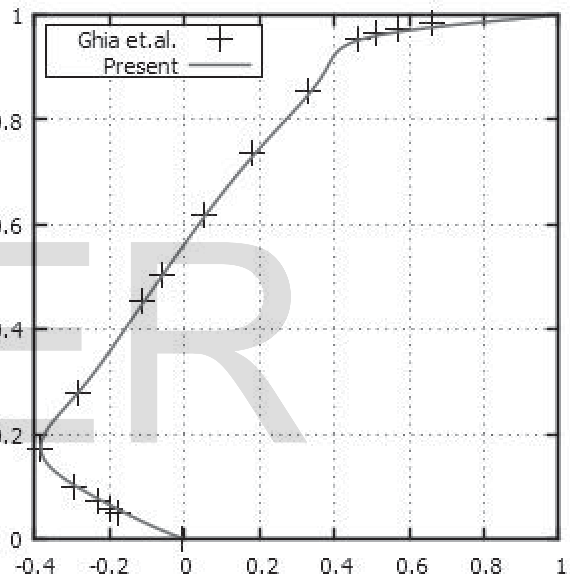

Figure 1. Horizontal component of velocity
along the middle vertical plane of the cavity for Re = 1000

2. Mixed convection in inclined channels with heat sources

The second problem selected for benchmarking is reported in literature by Menon et. Al [7] and is selected to check the capability of the software to predict mixed convection flows. The geometry is an inclined rectangular channel having constant-flux heat source of finite length on the lower surface. A single heat source is provided on one side of the channel wall and rest of the wall is kept adiabatic. Opposite wall is maintained at constant temperature. The ranges for the Reynolds number, the Grashof number, and the inclination angle are, $1 \le \mathrm{Re} \le 10$, $10^3 \le \mathrm{Gr} \le 10^5$, and $0° \le \gamma \le 90°$ respectively. The Figure 2 shows the temperature distribution along the heat source module obtained from the present simulation and the reported one for Gr=10^5, Re = 1, 5, 10 and angle 0°. A good agreement is obtained and so it is proved that the software and the methodology followed is capable of accurate prediction of mixed convection flows.

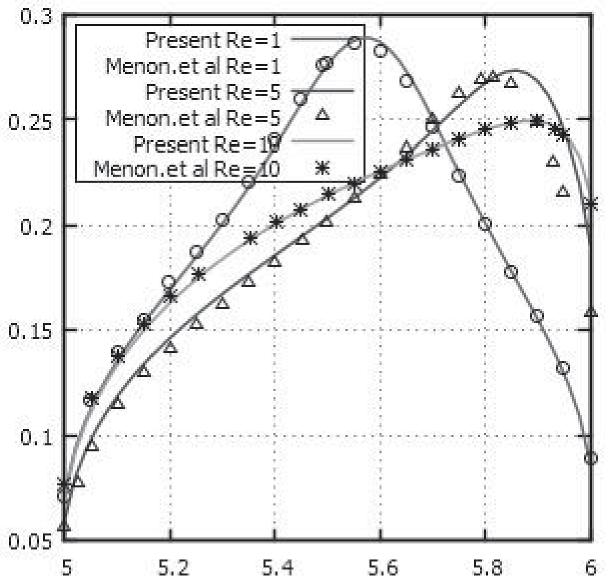

Figure.2 Module temperature distributions for Gr = 10^5, γ = 0° and Re = 1, 5, and 10

3. Combined heat transfer of Radiation and natural convection in a square cavity containing participating gases.

In order to validate the radiation model, a benchmark problem available in the literature is selected for validation. Combined heat transfer of radiation and natural convection in a square cavity is containing participating gases under normal room conditions [8].

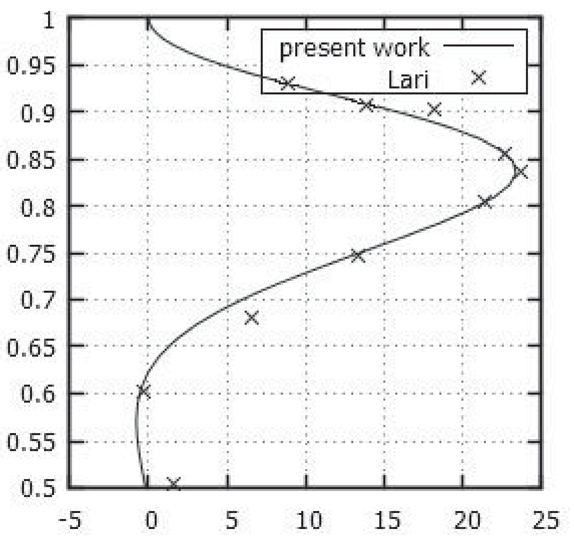

Figure.3. U-velocity along the Y-axis at x = L/2

The U-velocity distribution along Y-axis at x = L/2 for optical thicknesses=1 is shown in Figure.3. Surface radiation effect on the natural convection flow is clearly established from this validation. The comparison with benchmark solution is and the deviation is found to be in an acceptable level.

C. *Detailed description of the problem under consideration.*

A numerical model is formulated to solve the two-dimensional heat transfer in vertical channel with five discrete heat sources using FLUENT. The schematic of the geometry is shown in Figure 4. One side of the channel wall has five discrete heat sources mounted and the rest of the wall is kept adiabatic. The opposite wall is maintained at constant temperature. The geometrical parameters under consideration are taken as, (1) Width of channel = H, (2) Length of channel = 23H, (3) Length of Heat source = H, (4) Distance to the first heat source from inlet = 5H, (5) Distance between heat sources = H and (6) Distance from the last heat source to exit of the channel = 9H.

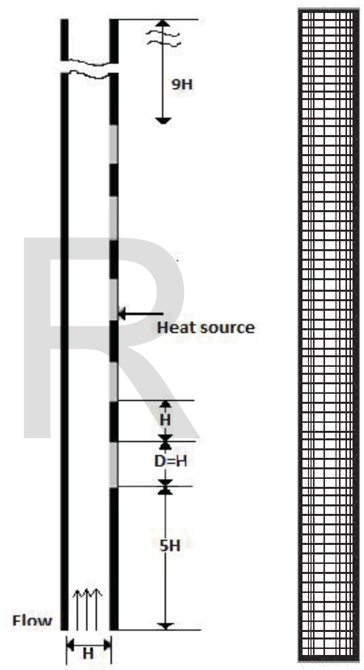

Figure.4. Schematic diagram and the computational grid

The ranges for the Reynolds number and Grashof number are $10 \le \mathrm{Re} \le 100$ and $10^3 \le \mathrm{Gr} \le 10^7$ respectively. The computational mesh used is as shown in Fig. 7. The flow is assumed to be laminar incompressible and the fluid properties assumed are; Prandtl number is kept constant at 0.71 and it remains nearly constant over a wide range. The density ρ = 1kg/m^3, viscosity μ =0.001 N-s/m^2, thermal conductivity k = 0.001W/(m.k) and the heat flux is in the order of 100 W/m^2. Grashof number the range of $10^3 \le Gr \le 10^7$ and Reynolds number various Re=10, 50, 100, 250, 500, 750, 1000. Acceleration due to gravity g is calculated using Grashof number equation and other values are kept constant. Aspect ratio is the geometrical parameter representing the relation between width and heat sources. Aspect ratio (AR) is defined as the ratio between the length of the heat sources and width of channel and it is varying from 1 to 2. The SIMPLE solution algorithm is used to handle the pressure–velocity coupling.

D. Grid independence study

A grid independence study is conducted to get an appropriate grid size for the present numerical investigation. In order to show that the present results are independent of mesh size, seven different mesh sizes are examined, as depicted in Table 1 and figure 5. First a relatively coarse mesh is generated using GAMBIT around with size 100×230. Simulations are conducted on this grid and then grid is made fine slightly. Different grid sizes with the number of nodes varying from 2541 to 114381 be tested to check the grid independence.

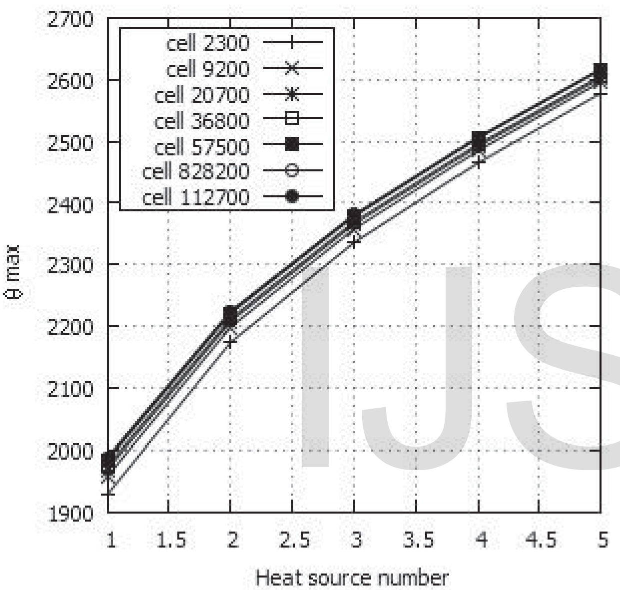

Figure.5 θ max with Heat source number in different grid.

Table.1 Comparison of different mesh size

	Number of cells	**Number of nodes**	**Percentage change in average temp with the previous case**
case1	10 X 230	2541	
case2	20 X460	9681	3.351189
case3	30 X 690	21421	1.320099014
case4	40 X 920	37761	0.702562965
case5	50 X1150	58701	0.06219017
case6	60 X1380	84241	0.283410418
case7	70 X1610	114381	0.201331138

The average value of temperature over the first heat source is selected and compared on each case. It can be seen that as the grid is refined, the difference between consecutive refinement levels reduces and reaches an acceptable minimum value at case number 6. This grid is selected and used for further solutions.

III. RESULTS AND DISCUSSION

The convection and radiation heat transfer flow inside fully open vertical channels with discrete heat sources using different thermal parameters and subsequent effects, finally settled to a result as explained below.

I. Case 1: Natural convection alone

The first part of the natural convection is analyzed based on aspect ratio. The thermal parameter selected is Grashof number 10^3 to 10^7. The aspect ratio is range from 1 to 2. Uniform heat fluxes are maintained in all the heat sources. Analysis is carried out for a total of 30 cases and the parameters θ_{max} and θ_{min} are selected for comparison due to their particular importance in the application level. Figure 6 (a) shows the variation of θ_{max} with aspect ratio over the fifth heat source. At higher values of Gr, it can be seen that the value of θ_{max} is almost independent of AR. Beyond Gr = 10^6, some noticeable variations in the value of θ_{max} with AR are present. As the Gr decreases below $10^{5,}$ the value of θ_{max} show a rapid raise and its dependence on AR dominates. The variation of θ_{min} is shown in figure 6 (b) and is found to be almost independent of aspect ratio in all the cases considered. The non-dimensional temperature θ are minimum on the first heat source because of the higher heat transfer rate due to the higher value of heat transfer coefficient and large temperature difference between the flow and the surface. Figure 7(a) shows the variation of θ_{max} with aspect ratio over different heat sources at Gr=10^5. The value of θ_{max} on the surface of the heat source increases almost linearly with AR. It is also observed that maximum value of temperature (θ_{max}) is found at the last (fifth) heat source while minimum in the first heat source. Figures 7(b) represent the dependence of average Nusselt number with aspect ratio for different values of Gr in first heat source. The minimum value of θ_{max} on the first heat source is caused by the highest value of the average Nu. At lower Grashof numbers, it can be seen that average Nu is independent of AR and it increases with increase in Gr and AR. Based on the above numerical results of the CFD analysis, a correlation between Nusselt number, Grashof number and aspect ratio is developed. A two variable power law approximation is used. The goodness of fit is checked and the agreement of the numerical data with the proposed correlation is represented in the form of a parity plot shown in (Figure 8). These correlations are based on 30 data points and have an index of correlation of 0.99 and percentage RMS error at 1%. This correlation are applicable to natural convective heat transfer in vertical open channels with 5 discrete heat sources having aspect ratio and Strength of convection have a strong dependence on aspect ratio and is independent of Gr.

$$\left.\begin{array}{l} Nu_1 = 9.63Gr^{0.18}AR^{0.72} \\ Nu_2 = 6.04Gr^{0.20}AR^{0.75} \\ Nu_3 = 5.19Gr^{0.20}AR^{0.76} \\ Nu_4 = 4.81Gr^{0.20}AR^{0.77} \\ Nu_5 = 4.61Gr^{0.20}AR^{0.78} \end{array}\right\} \text{for } (1 \le AR \le 2) \text{ and } (10^3 \le Gr \le 10^7)$$

The exponent of aspect ratio has some noticeable variations under the influence of heat source position while exponent of Gr remains a constant.

II. Case 2: Forced convection alone

In forced convection the non dimensional thermal parameter is selected as Reynolds number Re=10, 50, 100, 250, 500, 750, 1000. The aspect ratio is range from 1 to 2. . Pr=0.71. Figure 9(a) illustrates the variation of average Nu with aspect ratio in all the heat sources. Average Nu decreases when the heat sources number increase from inlet to outlet. From the analyses it is observed that the maximum Average Nu is in the first heat source, and in the minimum average Nu is in the last one (fifth heat source). Figures 9(b) shows the variation of average Nu with aspect ratio for different values of Re, and it is observed that as AR increases, average Nu increase almost linearly. These observations show that the forced convective heat transfer flow is extremely depended by the value of Reynolds number. At low values of Reynolds number, the convective heat transfer flow becomes weaker and the resulting slow fluid circulation reduces the forced convection heat transfer coefficient. Based on the numerical results of the CFD analysis, a correlation between Nusselt number, Reynolds number and aspect ratio is developed. A two variable power law approximation is used. The goodness of fit is checked and the agreement of the numerical data with the proposed correlation. These correlations are based on 42 data points and have an index of correlation of 0.99 and percentage RMS error at 1%. These correlations are applicable to forced convective heat transfer in vertical open channels with 5 discrete heat sources having aspect ratio $1 \le \mathrm{AR} \le 2$ and $10 \le \mathrm{Re} \le 1000$.

$$\left.\begin{array}{l} Nu_1 = 12.96Re^{0.36}AR^{0.58} \\ Nu_2 = 9.59Re^{0.35}AR^{0.58} \\ Nu_3 = 8.55Re^{0.35}AR^{0.60} \\ Nu_4 = 8.16Re^{0.34}AR^{0.61} \\ Nu_5 = 7.98Re^{0.33}AR^{0.62} \end{array}\right\} \text{for } (1 \le AR \le 2) \text{ and } (10 \le Re \le 1000)$$

It is clear that Nu is an increasing function of both Re and AR and these correlations dictates the strong dependence of forced convection on Re and aspect ratio. The exponent of aspect ratio have some noticeable variations is under the influence of heat source position.

III. Case 3: Mixed convection

Mixed convection heat transfer analyses have been carried out in the same geometrical configuration similar to the previous cases. Simulations were conducted for different range of Grashof number 10^3 to 10^7, aspect ratio 1 to 2 and Reynolds number 10, 50, 100, 250, 500, 750 and 1000. One more non-dimensional number, Richardson number (Ri) defined as Gr/Re^2 is introduced to reduce the number of variables. Figure 10(a) show the θ_{max} with Richardson number in all heat sources for aspect ratio 1. The θ_{max} is found to be increasing with Richardson number and it is also observed that maximum value of temperature (θ_{max}) is present at the last (fifth) heat source while the minimum (θ_{min}) in the first heat source. The variation of θ_{min} with Richardson number is shown in figure 10(b). In both the cases Richardson number is based on constant value of Gr $=10^3$ and varying Re from 10 to 1000. Figure 11(a) gives the variation of average Nusselt number with Richardson number for different aspect ratios at the first heat source. The Richardson number is calculated based on constant value of Gr$=10^5$ and varying Re from 10 to 1000. The value of Nu is found to be increasing with Ri for all aspect ratios. The observations show that the highest average Nu is present for aspect ratio 2 and minimum is for aspect ratio 1. Figure 11(b) shows the variation average Nusselt number with Richardson number in different heat sources for aspect ratio 1. At this time Ri is based on constant value Re 10 and Gr varying from 10^3 to 10^7. It is found that value of average Nu decreases while affect from heat source 1 to 5 (in the ascending order of their position). This may be due to fact that as moving from bottom to top, their convective heat transfer coefficient decrease, leading to the reduction in the value of Nusselt number. So it can be concluded that, position of the heat sources play a vital role in their cooling. Similar trends have been found for all aspect ratios. It is interesting to note that average Nu in almost all heat sources become equal at Ri = 10000 for Re=10. Three region of heat transfer have been delineated from Nu Vs Ri graph; Richardson number ≤ 10000 leads to forced convection, Richardson number ≥ 10000 leads to natural convection and the pure mixed convection flow at Richardson number 10000.

$$\left.\begin{array}{l} Nu_1 = 16.39Gr^{0.07}Re^{0.21}AR^{0.64} \\ Nu_2 = 11.66Gr^{0.09}Re^{0.19}AR^{0.65} \\ Nu_3 = 9.83Gr^{0.09}Re^{0.18}AR^{0.65} \\ Nu_4 = 9.29Gr^{0.09}Re^{0.17}AR^{0.68} \\ Nu_5 = 8.81Gr^{0.09}Re^{0.17}AR^{0.67} \end{array}\right\} (1 \le AR \le 2),$$

$$\text{for } (10^3 \le Gr \le 10^7) \;\&\; (10 \le Re \le 1000)$$

Based on the above numerical results of the CFD analysis, a correlation between Nusselt number, Grashof number, Reynolds number and aspect ratio is developed. A three variable power law approximation is used. The goodness of fit is check. These correlations are set from 210 data points and have an index of correlation of 0.99 and percentage RMS error at 1%. These correlations are valid to mixed convective heat transfer in vertical open channels with 5 discrete heat sources having aspect ratio $1 \le AR \le 2$, Grashof number $10^3 \le Gr \le 10^7$ and Reynolds number $10 \le Re \le 1000$. The correlations show that heat source position can control the convective heat transfer in mixed convection. It is noted that coefficient of Gr remains constant for all heat sources, i.e., there is a linear dependence of Nu with Gr in mixed convection.

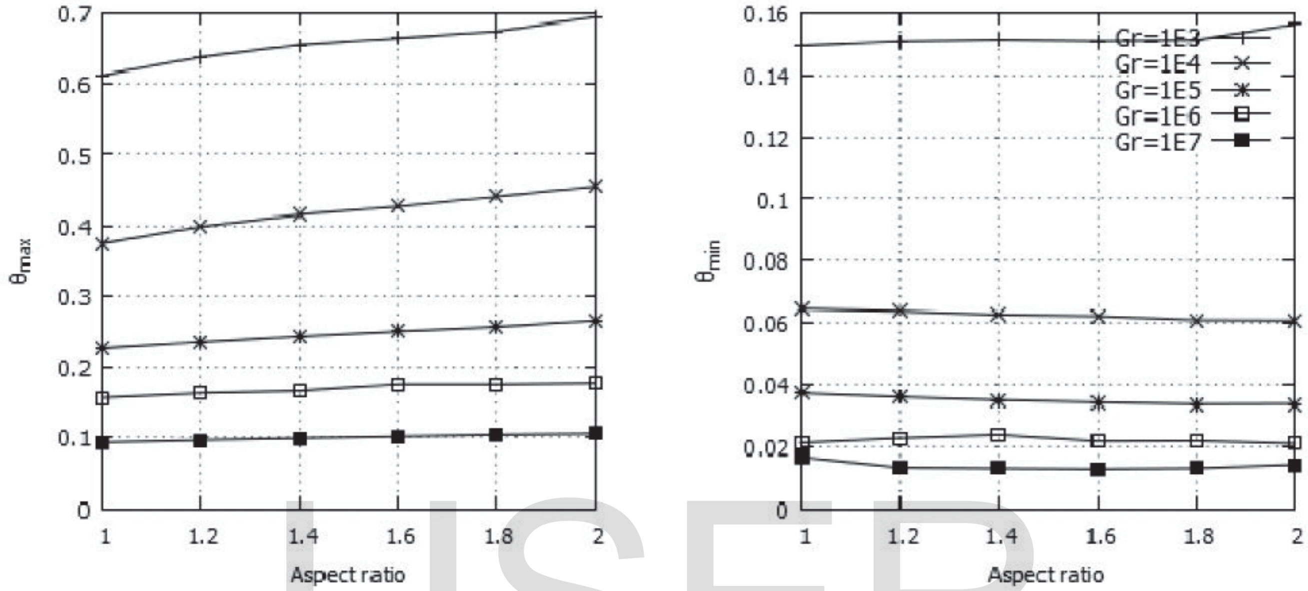

Figure 6. Variation of (a) θ_{max} with aspect ratio in heat source 5 and (b) θ_{min} with aspect ratio in heat source 1

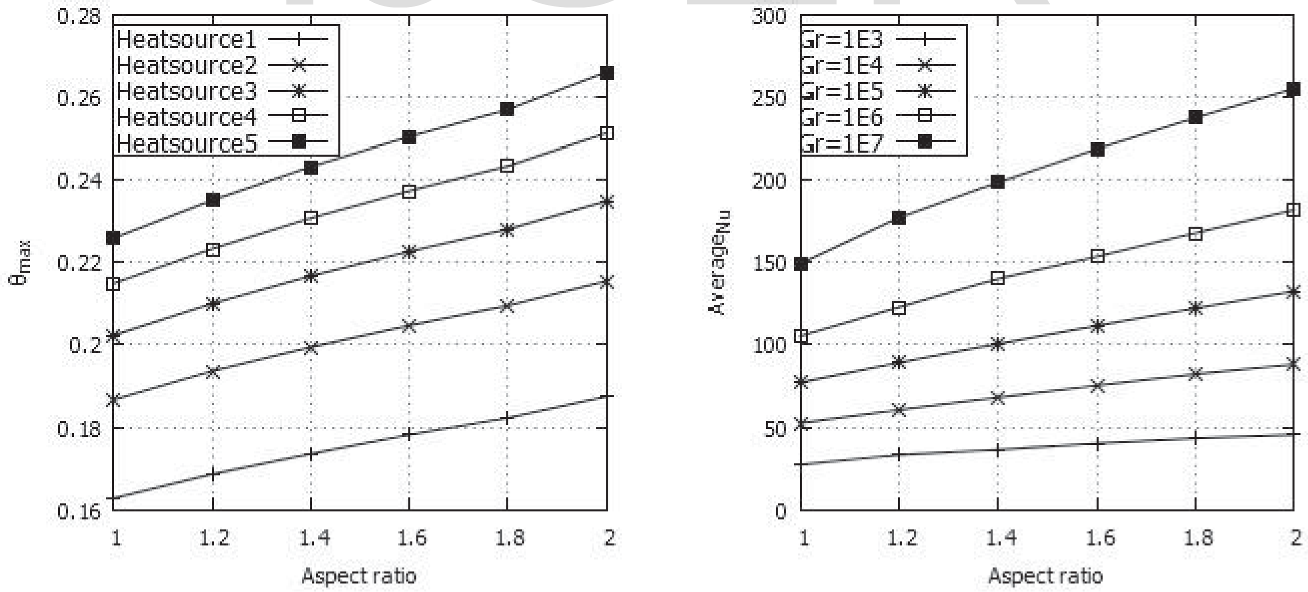

Figure 7. Variation of (a) θ_{max} with aspect ratio over different heat sources and (b) average Nu with AR in heat source 1.

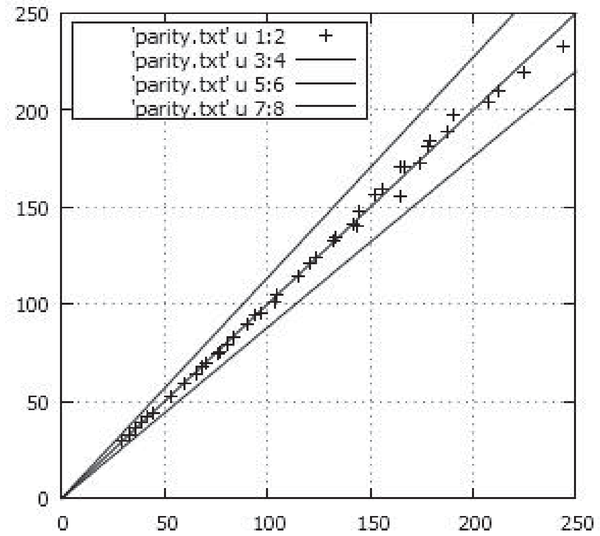

Figure8. Parity plot showing the goodness of fit.

IV. Case 4: Effect of surface radiation

After completing the study on the effects of various geometrical and physical parameters on the convective cooling inside a vertical channel with discrete heat sources, it is decided to extend the study by considering the effects of surface radiation on the mixed convection heat transfer. Analysis is conducted for a single aspect ratio, AR = 1. In this work, only a single parameter of the standard model, the optical thickness (α) is varied while keeping all others constant. DTRM model is chosen for the present simulations. Analysis is carried out for different Richardson number with constant aspect ratio=1. Richardson number is calculated from Grashof number Gr=10^5 and a range of Reynolds number, Re= 10, 50, 100, 250, 500, 750, 1000. Figure 12(a)illustrates the variation of θ_{max} with Richardson number in fifth heat source, under the presence of a non-participating medium and participating medium with different optical thickness values.

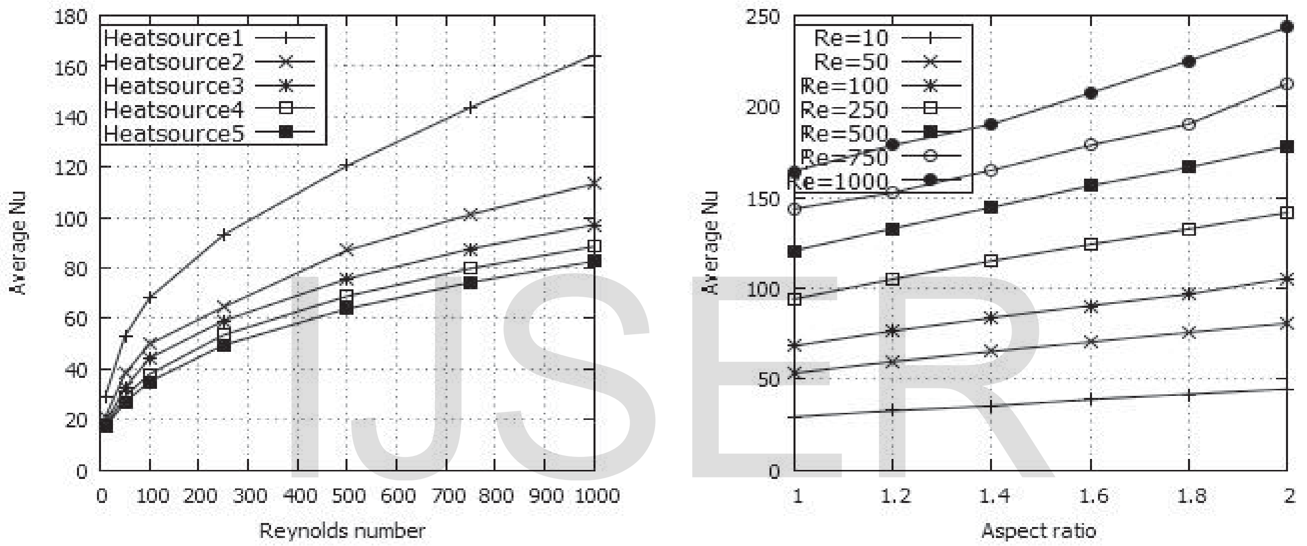

Figure 9. Variation of (a) average Nu with Reynolds number in all the heat source and (b) avrage Nu with Aspect ratio in heat source 1.

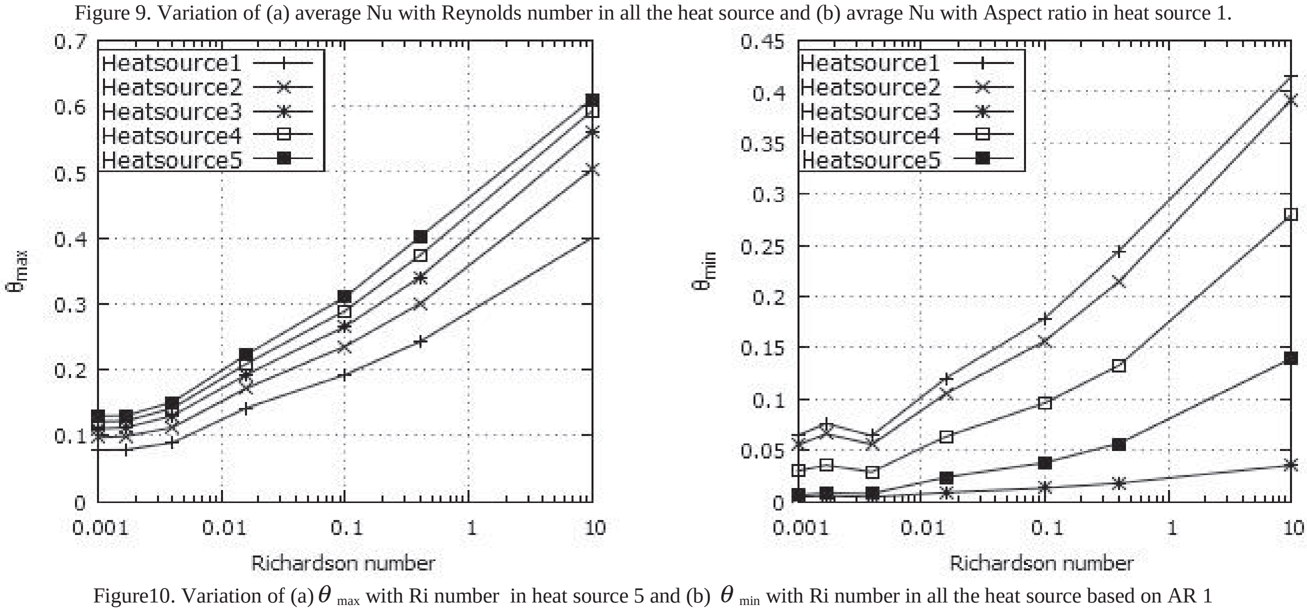

Figure10. Variation of (a) θ_{max} with Ri number in heat source 5 and (b) θ_{min} with Ri number in all the heat source based on AR 1

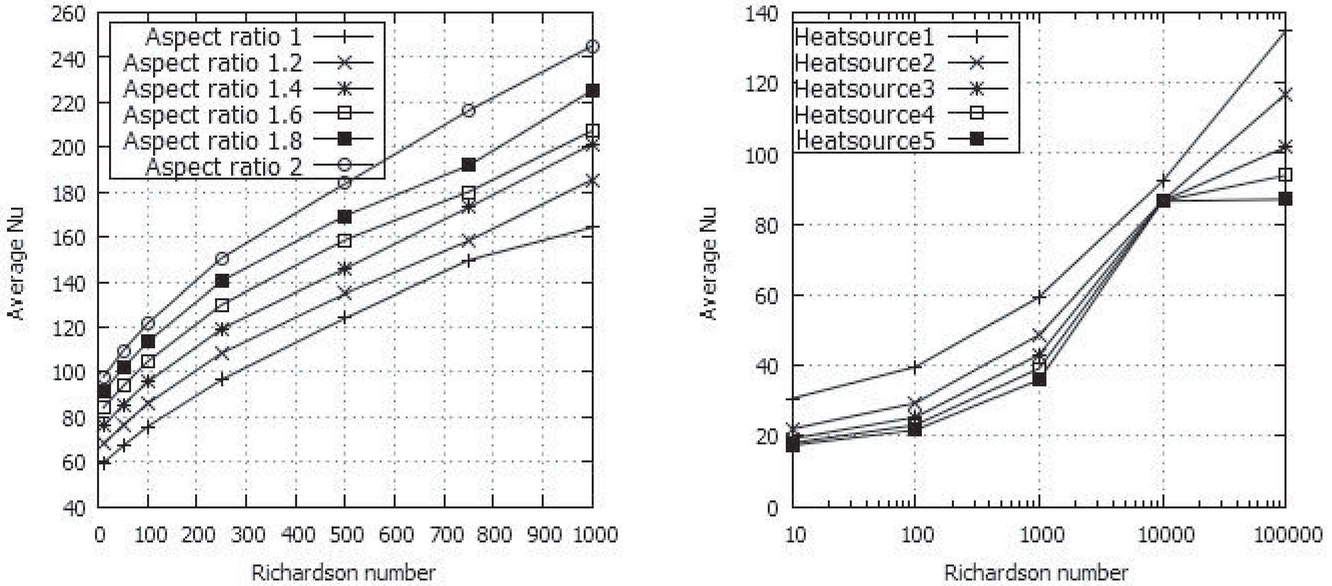

Figure 11. Variation of (a) average Nu with Richardson number in heat source 1 and (b) avrage Nu with Richardson number in the entire heat source

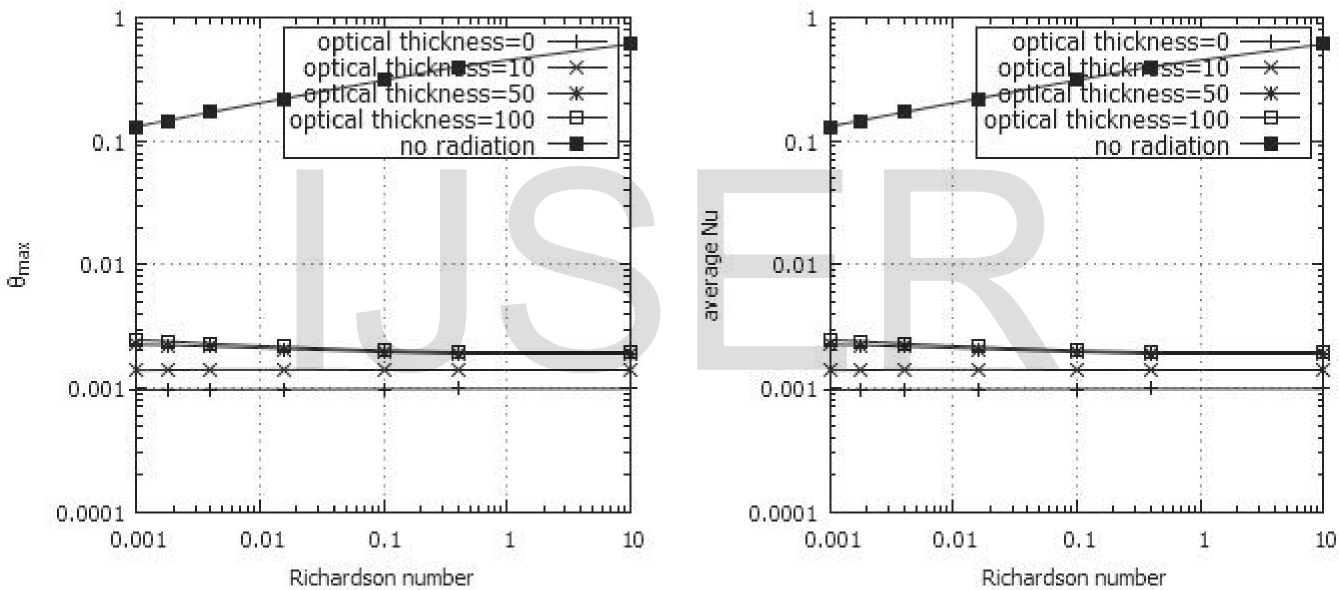

Figure.12 Variation of (a) θ_{max} with Richardson number and (b) Variation of average Nu with Richardson number

Finally analysis is performed to simulate pure mixed convection without considering the radiation model. Figures (b) show the variation of average Nu with Richardson number for the same cases mentioned above. Both figures are almost similar result because the surface radiation effect will influence the large rate of heat transfer in the channel. For optical thickness (α) = zero (ie., non-participating medium) the value of θ_{max} and the average Nu are found minimum. It is clear that with increase in the value of optical thickness the strength of the convective heat transfer flow increases and also found that, surface radiation effects are capable of causing significant variations in rate of heat transfer inside the channel.

IV. CONCLUSIONS

The following observations have been made from the numerical study. Some salient conclusions of the present study are summarized as follows:

- In all the cases, first heat source has minimum non-dimensional temperature (θ). This is due to the maximum convective heat transport in this area by the large temperature difference between the heat source and ambient air. The last heat source has the minimum convection heat transfer and so maximum non-dimensional temperature because of the smaller temperature difference.

- Natural convective heat transfer is analyzed with varying $1 \leq AR \leq 2$ and $10^3 \leq Gr \leq 10^7$ and correlations are developed. The Nusselt number correlations point out that, strength of convection has a strong dependence on aspect ratio and is independent of Gr.

- Forced Convection heat transfer is simulated with Re =10, 50, 100, 250, 500, 750, 1000 and $1 \leq AR \leq 2$. It is found that convective heat transfer increases with increase in Reynolds number. The Nusselt number correlations concludes that there is a strong dependence of forced convection on Re and aspect ratio.

- Mixed convection heat transfer is analysed in the range $1 \leq AR \leq 2$ and $0.001 \leq Ri \leq 10^5$ and it is interesting to find that average Nu in almost all heat sources become equal at Ri = 10000 and Re=10.

- Three region of heat transfer have been delineated from Nu Vs Ri graph; Richardson number ≤ 10000 leads to forced convection, Richardson number ≥ 10000 leads to natural convection and the pure mixed convection flow at Richardson number 10000.

- It is found that, surface radiation effects are capable of causing significant variations in rate of heat transfer inside the channel. Further studies in this area are required to reveal the flow characteristics.

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